Morrey smoothness spaces: A new approach

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Abstract

In the recent years so-called Morrey smoothness spaces attracted a lot of interest. They can (also) be understood as generalisations of the classical spaces $A^s_{p,q}(\mathbb{R}^n)$, $A \in \{B, F\}$, where the parameters satisfy $s \in \mathbb{R}$ (smoothness), $0 < p \leq \infty$ (integrability) and $0 < q \leq \infty$ (summability). In the case of Morrey smoothness spaces additional parameters are involved. In our opinion, among the various approaches at least two scales enjoy special attention, also in view of applications: the scales $\mathcal{A}^s_{u,p,q}(\mathbb{R}^n)$, with $\mathcal{A} \in \{\mathcal{N}, \mathcal{E}\}$, $u \geq p$, and $A^{s,\tau}_{p,q}(\mathbb{R}^n)$, with $\tau \geq 0$.

We reorganise these two prominent types of Morrey smoothness spaces by adding to (s, p, q) the so-called slope parameter ϱ, preferably (but not exclusively) with $-n \leq \varrho < 0$. It comes out that $|\varrho|$ replaces n, and $\min(|\varrho|, 1)$ replaces 1 in slopes of (broken) lines in the $(\frac{1}{p}, s)$-diagram characterising distinguished properties of the spaces $A^s_{p,q}(\mathbb{R}^n)$ and their Morrey counterparts.

Our aim is two-fold. On the one hand we reformulate some assertions already available in the literature (many of them are quite recent). On the other hand we establish on this basis new properties, a few of them became visible only in the context of the offered new approach, governed, now, by the four parameters (s, p, q, ϱ).

The talk is based on joint work with Hans Triebel (Jena).