Supercaloric functions for the parabolic p-Laplace equation

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Abstract

We discuss a generalized class of supersolutions, so-called p-supercaloric functions, to the parabolic p-Laplace equation

$$\frac{\partial u}{\partial t} - \operatorname{div}(|\nabla u|^{p-2}\nabla u) = 0, \quad 1 < p < \infty.$$

This class consists of lower semicontinuous functions that are finite in a dense set and satisfy a parabolic comparison principle. Their properties are relatively well understood in the slow diffusion case $p > 2$, but little is known in the fast diffusion case $1 < p < 2$. Supercaloric functions form a similar basis for a nonlinear parabolic potential theory as in the classical case of the heat equation with $p = 2$. The leading example of a p-supercaloric function with a point singularity is the Barenblatt solution, which corresponds to the fundamental solution. In the fast diffusion case so-called friendly giant and other examples constructed by separation of variables are included in the theory. We discuss existence, structural, convergence and Sobolev space properties of p-supercaloric functions. Several characterizations are given and connections to measure data problems are explained.