Strongly nonlinear multiplicative inequalities with elliptic operators

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Abstract

In 2012, together with Jan Peszek, we obtained the following inequality:

$$\int_{(a,b)} |f^{'}(x)|^q h(f(x)) dx \leq C \int_{(a,b)} \left(\sqrt{|f^{''}(x)\mathcal{T}_h(f(x))|} \right)^q h(f(x)) dx,$$

as well as its Orlicz variants, where $\mathcal{T}_h(f)$ is certain transformation of function f with the property $\mathcal{T}_{\lambda^\alpha}(f) \sim f$, generalizing previous results in this direction due to Mazja.

We will discuss further developements of this inequality, focusing on its multidimensional variant obtained recently with Tomas Roskovec and Dalmil Pesa:

$$\int_{\Omega} |\nabla f(x)|^q h(f(x)) dx \leq C \int_{\Omega} \left(\sqrt{|Pf(x)\mathcal{T}_h(f(x))|} \right)^q h(f(x)) dx,$$

which involves the elliptic operator P. We will also present some applications of such inequalities to the regularity theory for the nonlinear PDE's of elliptic type.

The talk will be based on the the chain of my joint works obtained together with Katarzyna Pietruska-Pałuba, Jan Peszek, Katarzyna Mazowiecka, Tomasz Choczewski, Alberto Fiorenza and Claudia Capogne, Tomas Roskovec and Dalmil Pesa.